When to stop searching in the future something we hope to be better?

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Saona, R.[1], Correa, J.[1] & Ziliotto, B.[2]

Universidad de Chile[1], Dauphine University[2]

Industrial applications

How to...

Apply price discrimination to sell my products?

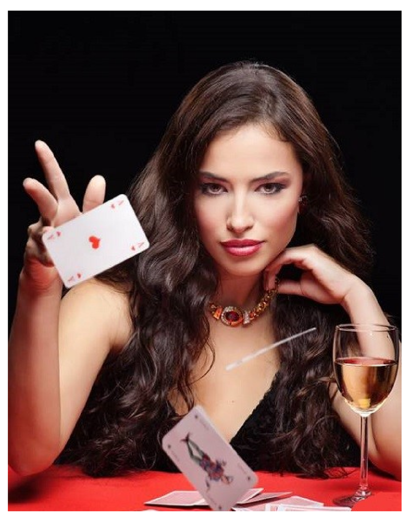
Set different prices for different consumers based on purchase history?

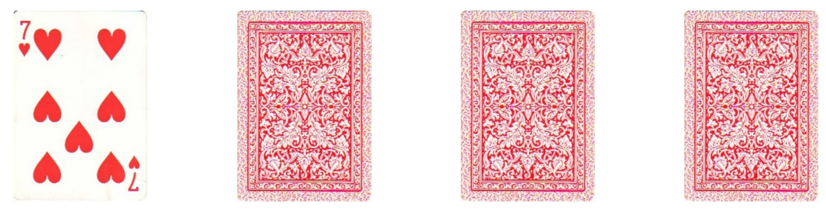
Determine the personalized reserve prices for Google ads?

Facts

The online data provider Lexis-Nexis sells to virtually every user at a different price.

Orbitz online travel agency found that people who use Mac computers spent as much as 30% more on hotels.





Myerson's Auction → (simpler; no strategic behaviour) → **Posted Price Mechanisms** ↔ (equivalent) ↔ **Prophet Secretary**

Description

A gambler is faced with a stream of numbers, which are shown one by one. She must select only one value and only when it's first discovered.

The **goal** of the gambler is to maximize the expected value of what she picks, she knows the distributions in advance.

The **performance** measure is the worst case ratio between the expected value the gambler gets and what a prophet, that sees all the realizations in advance, gets.

Objective

$$\inf_{F_1,\dots,F_n} \sup_T \frac{\mathbb{E}(V_{\sigma_T})}{\mathbb{E}(\max\{V_1,\dots,V_n\})}$$

σ : Random order

T: Strategy

$(V_i)_{i\in[n]}$: Positive independent random variables

Previous works

Adversarial order (σ is chosen by an oponent)

Krengel, U., & Sucheston, L.
Semiamarts and finite values.

IID case ($V_i \sim F$)

Correa, J., Foncea, P., Hoeksma, R., Oosterwijk, T. & Vredeveld, T.
Posted price mechanisms for a random stream of customers.

Constant threshold (T restricted to a subclass)

Ehsani, S., Hajiaghayi, M., Kesselheim, T. & Singla, S.
Prophet Secretary for Combinatorial Auctions and Matroids.

Blind strategies

It is defined by $\alpha : [0,1] \to [0,1]$ through the following procedure:

1) Compute the thresholds $\tau_1, \dots, \tau_n$ such that

$$\mathbb{P}(\max\{V_1,\dots,V_n\} \le \tau_i) = \alpha(i/n)$$

2) Take the i-th presented value if it surpasse τ_i .

Theorem:
Exists a α with a performance of **at least** 0.669, and every blind strategy performes **less than** 0.675.

Theorem:
Every non-adaptive strategy performes **less than** 0.734

Optimal Solution

Given by **Dynamic Programming**, it can be stated as follows

"Take the first variable that surpases the expected value you would get by leaving it."

Comparison

Number of thresholds: n versus n!

Performance: Blind strategies achieve at least 90% of the optimal.

Definition: Encoded in α versus local definition.

Decisions: Stochastic tie breaking versus deterministic choices.

Good to know

Schur-convexity is a key conept in our analysis and allows us solve the following problem.

$$(P)\begin{cases} \max_x & f(x) \\ s.t. & \sum_{i\in[n]} x_i = s_0 \\ & x \in A\,. \end{cases}$$

where A is a permutation symmetric set.

Open questions

How good is the optimal strategy?

How to take advantage of choosing a **specific order**?

Can we deal with **less information** than the distribution of each option?

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